Experimentation

October 27, 2015

Generalization

- We want to know how a predictor h will perform ***in general***.
- What do you mean ***in general***?
  - "Average" behavior for all possible inputs (e.g., sentences, DNA sequences, corpora, …)***, even the ones we don't have in our training/test data***

$$\mathbb{E}_{p(\boldsymbol{x},\boldsymbol{y})}\text{cost}(h(\boldsymbol{x}),\boldsymbol{y})$$

Experimentation

- That expectation can’t be computed
  - Rather than looking at all possible inputs (maybe infinite! Maybe huge!), look at a **representative sample** of inputs
  - Make **inferences** from these experiments about the rest of the **population**
  - Rough idea: if we do well on a representative sample, we will do well on the whole population
- Mathematics can show provide conditions under which these inferences will be true with high probability

Standard Methodology

- We want to compare at two predictors h and h' that differ in a well-defined way
  - Data used to train them
  - Algorithm used to train them
  - Training objective (e.g., conditional vs. joint)
  - Feature set used
  - Inference method (e.g., exact vs. approximate)
  - Decoding objective (e.g., MAP vs. MBR)

Which predictor is better?

That is, we would like to know whether:

$$\mathbb{E}_{p(\boldsymbol{x},\boldsymbol{y})}[\text{cost}\left(h(\boldsymbol{x}),\boldsymbol{y}\right)] < \mathbb{E}_{p(\boldsymbol{x},\boldsymbol{y})}[\text{cost}\left(h'(\boldsymbol{x}),\boldsymbol{y}\right)]$$

Unfortunately, we cannot generally know this! ☹

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But we can know the following: ☺️

Test set: $\mathcal{T} = \{\boldsymbol{x}_i^*, \boldsymbol{y}_i^*\}_{i=1}^{N^*}$

$$\frac{1}{N^*}\sum_{i=1}^{N^*} \text{cost}\left(h(\boldsymbol{x}_i^*), \boldsymbol{y}_i^*\right) < \frac{1}{N^*}\sum_{i=1}^{N^*} \text{cost}\left(h'(\boldsymbol{x}_i^*), \boldsymbol{y}_i^*\right)$$

Other Scenarios

- We may want to compare more than two predictors
- We may want to compare more than one cost function
- We may be working with cost functions that are defined at the corpus level
  - F-measure, precision, recall, BLEU, ROUGE, etc.

Held-Out Test Sets

- **Number one rule:** Keep your training data out of your test data
- If this sounds simple, it is anything but
  - Selecting hyperparameters by looking at the test set scores
  - Every year ***many*** (most?) papers are published that violate this!
- Standard recipe
  - **Training data** (possibly further subdivided into training & tuning)
  - Held-out **development data** [use while developing system]
  - Blind **test data** [for publication only]

Held-Out Test Sets

- Years of experimentation with “blind” test sets means they aren’t “blind” any longer!
- Strategies for dealing with this
  - Periodic creation of new test community sets
  - Fix all parameters of development data, report on held-out test data **[publication bias]**
  - Cross-validation

- **I’ll say it again:** Using held-out test data is the **single most important thing you can do** to ensure your experiments give generalization insight

Generalization: Cross Validation

- Sample train/dev/test data from D
- k-fold cross validation
  - Select k train/dev/test splits
- In the limit: k=N, “leave-one-out” CV
  - If you have N training instances, run N experiments training on N-1 instances
- Pros
  - More statistical power
  - Better use of limited data resources
- Cons
  - Computationally expensive
  - Not terribly common in structured prediction

Oracles and Upper Bounds

- What is the best possible performance knowing something about the test set?
  - Up to, and including, the test set!
- Examples
  - Tuning hyperparameters or parameters on the test set
  - Using gold standard parse trees or NER labels for a downstream information extraction task
- Answers a different question than generalization: does my model have adequate “capacity”?

Back to Generalization

- Is held-out data enough?
- How many samples do we need to make reliable inferences?
  - If you need to detect big differences, you need fewer samples
  - If you need to detect small differences, you need big samples
  - If you do lots of similar experiments looking for an effect, you're more likely to hit one "by chance"- can we control for this (false discovery)
- This brings us to…

Statistical Hypothesis Testing

- **Statistical predictors != statistical evaluation**
  - You can do statistical evaluation of non-statistical predictors!
- Hypothesis testing in one sentence: How likely is that the behavior we're seeing is due to chance?
- **Hypothesis testing is not magical**
  - *p*-values are not the probability your claim is wrong
  - At best, you find out what is the probability some pattern of results is due to chance
    - If the your results unlikely due to chance, this **does not mean** the hypothesis you formulated was true; converse is also true

Statistical Hypothesis Testing

- Formulate a **null hypothesis** H_0
  - Skeptical perspective: e.g., two experimental scenarios are the same
- Set a threshold with which we reject the null hypothesis, usually $\alpha \in \{0.05, 0.01, 0.001\}$
- What is the probability of the experimental observations, assuming the null hypothesis?
  - If $p < \alpha$, then we can reject H_0

Parameters & Statistics

$$u_i \sim U_i, \quad i = [1, N]$$

$$v_i = v(u_i), \quad (\text{ie., } v_i \sim V)$$

The **mean** (a *parameter*) is **not** a random variable; it is a **real number**.

$$\mu_V \doteq \mathbb{E}_{p(u)}[v(u)] = \int v(u) \cdot p(u) du$$

The **sample mean** (a *statistic*) is a function of $\mathbf{u}$, and therefore is a **random variable**

$$\hat{\mu}_V = \frac{1}{N} \sum_{i=1}^{N} v_i$$

Sampling Distribution

- A statistic, e.g. our sample mean

$$\hat{\mu}_V = \frac{1}{N}\sum_{i=1}^{N} v_i$$

  is a **random variable**.

- What distribution is it drawn from, i.e. can we say something about the following?

$$\hat{\mu}_V \sim \mathrm{Distribution}(\boldsymbol{\theta})$$

Sampling Distribution

- Under some weak assumptions, a central limit theorem tells us

$$\hat{\mu}_V \sim \mathcal{N}\left(\mu_V, \frac{\sigma_V^2}{N}\right)$$

- **This is an awesome result!** As *N* gets bigger, the expected deviation from the parameter of interest drops.

Standard Error

- What is the standard deviation of the sample mean?

σ_V parameter of global population

$\sigma_{\hat{\mu}_V}$ parameter of sampling distribution

$$\sigma_{\hat{\mu}_V} = \frac{\sigma_V}{\sqrt{N}}$$

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$\hat{\sigma}_V$ statistic: the sample standard deviation

$$\hat{\sigma}_V = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(u_i - \hat{\mu}_i)^2}$$

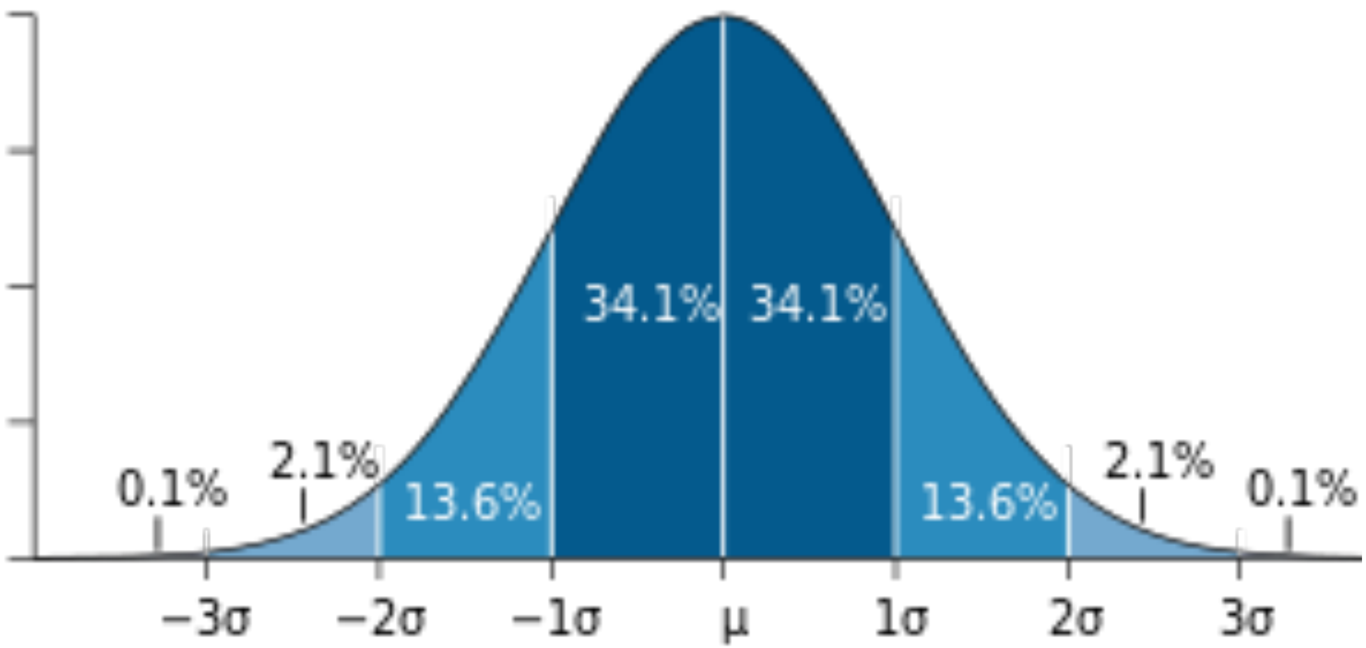
Standard Error

- We can now state the standard error

$$\hat{\sigma}_{\mu_V} = \frac{\sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(u_i - \hat{\mu}_i)^2}}{\sqrt{N}}$$

- This idea of replacing the true distribution (which we cannot know) with samples is the same thing we did with Monte Carlo techniques.

Standard Deviations



Other Parameters/Statistics

- Any ***generalized mean***:
  - min, median, …, max
- Proportions
  - proportion of a population for which property P holds
- Other functions
  - BLEU score, F-measure, word error rate…
- Except for proportions, these statistics don't have a closed form of the standard error

Bootstrap (Efron, 1979)

- Monte Carlo technique to estimate standard error of some statistic $\hat{\theta}_V$
- We have a sample of *N* draws from *U*

$$\mathbf{u} = (u_1, u_2, \ldots, u_N)$$

- For i=1 to *B*, resample *N* times from the empirical distribution of $\mathbf{u}$

$$\mathbf{u}^{(i)} = (u_1^{(i)}, u_2^{(i)}, \ldots, u_N^{(i)})$$

- From the sequence of bootstrap samples estimate the standard error

$$\hat{\sigma}_{\theta}^{(boot)} = \sqrt{\frac{1}{B-1}\sum_{i=1}^{B}\left(\hat{\theta}_{V,\mathbf{u}^{(i)}} - \frac{1}{B}\sum_{j=1}^{B}\hat{\theta}_{V,\mathbf{u}^{(i)}}\right)^{2}}$$

$$= \frac{\sqrt{\sum_{i=1}^{B}\left(\hat{\theta}_{V,\mathbf{u}^{(i)}} - \frac{1}{B}\sum_{j=1}^{B}\hat{\theta}_{V,\mathbf{u}^{(i)}}\right)^{2}}}{\sqrt{B-1}}$$

$$\sigma_{\theta} \approx \hat{\sigma}_{\theta} \approx \hat{\sigma}_{\theta}^{\mathrm{boot}}$$

(When $\theta_V = \mu_V$, $\hat{\sigma}_V = \sigma_V/\sqrt{N}$)